



Hale School

Mathematics Specialist

Test 3 --- Term 2 2019

Vectors

Name: _____

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Instructions:

- Calculators are allowed
 - 1 page of external notes are allowed
 - Duration of test: 45 minutes
 - Show your working clearly
 - Use the method specified (if any) in the question to show your working (Otherwise, no marks awarded)
 - This test contributes to 7% of the year (school) mark
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1.

[2, 4 = 6 marks]

In the triangle OAB , $\vec{OA} = 3\vec{i} + 4\vec{k}$ and $\vec{OB} = \vec{i} + 2\vec{j} - 2\vec{k}$.

(a) Determine $\angle AOB$

(b) Determine \vec{OP} , where P is the point on AB such that OP is perpendicular to AB .

2.

[2, 3 = 5 marks]

Given the points $A \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$, $B \begin{pmatrix} -2 \\ 3 \\ -2 \end{pmatrix}$ and $C \begin{pmatrix} 0 \\ -1 \\ 4 \end{pmatrix}$, determine:

(a) The equation of the line passing through A and B

(b) The equation of the plane, Π , in normal form, passing through A, B and C.

3.

[5 marks]

A plane, Π , contains the line $\frac{2-x}{3} = \frac{y}{-4} = z+1$ and is parallel to $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

Find the cartesian equation of Π .

4.

[5 marks]

Determine the possible values of p and q if the system of equations

$$\begin{aligned}x - y + 2z &= 1 \\2x - 5y + 5z &= 9 \\3x + 3y + pz &= q\end{aligned}$$

has:

- (a) a unique solution,
- (b) no solution,
- (c) infinite solutions.

5.

[3, 4, 2 = 9 marks]

A model aircraft follows a circuit in the plane defined by $r = (3 \cos 4t)i - (2 \sin 4t)j$.

- (a) Determine the initial position of the aircraft and its direction of motion.
- (b) Determine the time(s) throughout the flight where the velocity of the aircraft is parallel to $i + j$.
- (c) Determine the distance travelled by the aircraft from $t = 1$ to $t = 2$.

6.

[4 marks]

Given the lines $r = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} + \lambda_1 \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix}$ and $r = \begin{pmatrix} 3 \\ 13 \\ -15 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

Determine whether the lines intersect or are skew. If they intersect, find the point of intersection. If they are skew, show this clearly.

7.

[6 marks]

If the line $r = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ is tangent to the sphere with equation

$$\left| r - \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \right| = a, \text{ determine the value of } a.$$

_____End of Test_____